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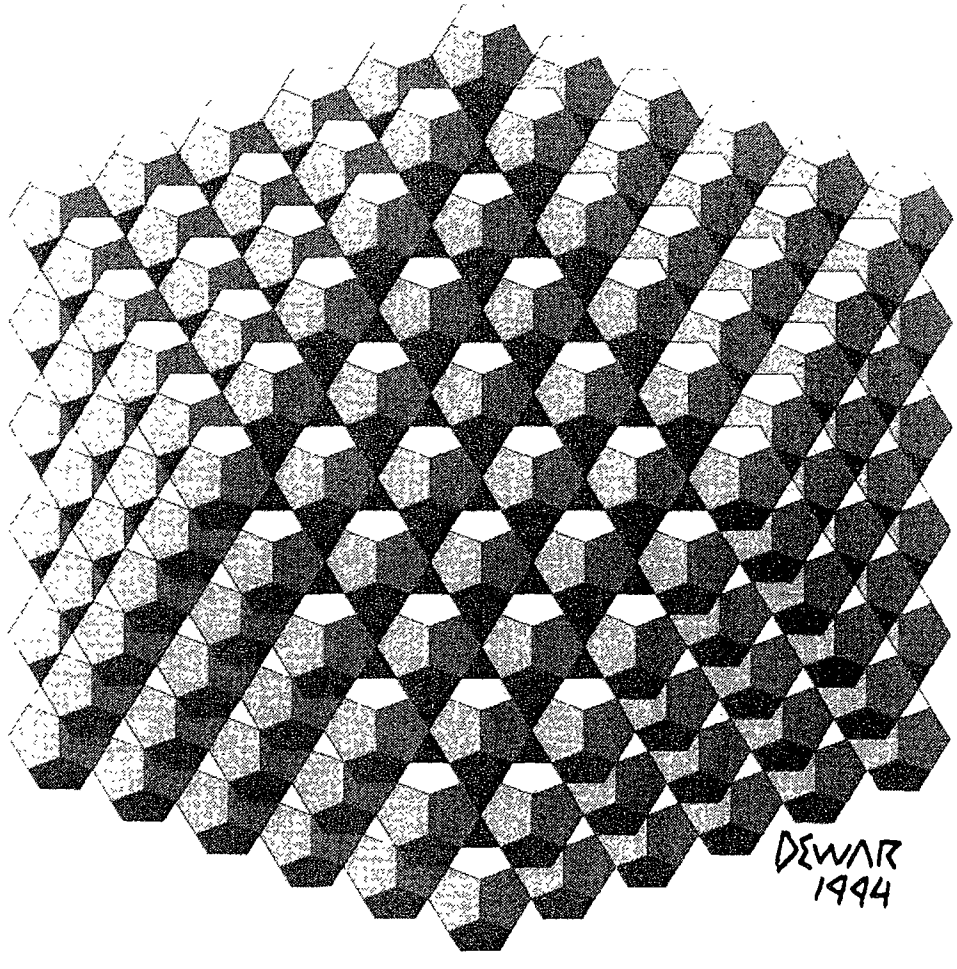

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Self Dual Space Lattices and Periodic Hyperbolic Surfaces

by M. Burt & A. Korren
Faculty of Architecture & Town Planning, Technion.-I.I.T.

The issue of space subdivision with continuous 2-manifold Surfaces was dealt over the years by many mathematicians, designers and morphologists of various sciences, because of its apparent importance, but never with an exhaustive search in mind.

A paper presented by the authors to Tsukuba Symposium (Nov.-1994[I]) had reported on such an effort. it was partial in its scope, and limited to that of Periodic (2-manifold) Hyperbolic Surfaces (P.H.S) which divide the space into two identical (congruent) subspaces.

Phenomenologically, the search was focused on construction of two identical (interwoven) dual spaces, graphically characterized by two dual space networks (Self Dual Lattices - S.D.L) and the 2-manifold surface which provides the continuous partition in between. The search was limited to such surfaces which represent a subgroup of all sponge structures.

The paper suggested a method of systematic exhaustive search of such surfaces and their symmetric-topological classification.

The method relied heavily on the "atomistic approach" of periodic spatial form, namely, an approach which enables to observe and analyze complex macro-characteristics through their representation on the "micro-elementary cells-simplex" level.

the application of the "atomistic approach" requires full understanding of the manifold relations between the "complex" periodic phenomenon and its "simplex" configuration, as well as the "subdivision-duplication mechanism" which transforms the one into the other.

The exhaustive search was based on a four-fold process :

(a) Exhaustive search and examination of all the possible E.P.R.s (Elementary Periodic Regious), based on the fact that all symmetry space groups are known and their number is finite

(b) Combinatorial, exhaustive identification of all the E.P.R.s, capable of containing 2-fold rotation axes which could rotate them into themselves.

(c) Combinatorial identification of all the possible different closed perimeters, within the resulting 2-fold axes lattices.

(d) Generation of all the possible different elementary (minimal) surface units within such closed perimeters and the related periodic surfaces, and their characterization and classification.

When all the different periodic surfaces (thus generated) were examined, it was revealed that, all in all, seven topologically different surfaces exist, identified already in 1969 by M. Burt[II].

With a deeper insight it was found that there is a weak point in the exercised method: it revolved on one intuitive (unconscious) assumption that the elementary minimal surface units under consideration could not be but simple; meaning, a product of a simulated dipping process of a closed wire perimeter in a soap solution, to produce a smooth, non-self-intersecting hyperbolic minimal surface.

This assumption was challenged by Korren in 1993[III] with a result that a door was opened to reveal a completely new array of surfaces-partitions and the associated self-dual space networks which characterize each of the two interwoven, complemementary subspaces (and appear to be an important contribution in its own right).

The conceptual breakthrough materialized with the realization that not only simply connected minimal surfaces are allowed (on the "simplex" level), but also surfaces corresponding to self-dual networks of a much higher complexity, characterized by higher genus and total curvature values.

In the light of these developments a need is arising for a redefinition and reformulation of the research goals, its direction of search and a conspicuous change of its methodology.

Redefinition of Research Goals and of Methodology.

It should be pointed out that there is a direct, one to one correspondence, between the hyperbolic (minimal or otherwise) partition surface (which subdivides the space) and the character and symmetrical nature of the two complementary subspaces, and between these two and the two mutually dual lattices representing them. In case the partitioned subspaces are identical

(congruent), the two mutually dual, complementary (reciprocal) lattices are identical (self-dual).*

This trichotomic phenomenon of the two complementary mutually dual and identical space lattices and the corresponding hyperbolic partition surface, which divides space into two identical subspaces, makes the exhaustive search of either as one and the same problem, and leaves it to convenience to determine which should be preferred and pursued first.

In view of the last development, there is an advantage in shifting the gravity center of the research and pursuing the self dual lattices pairs first, and that for the following reasons:

(a) The self dual lattices "could be extracted" directly from the relevant E.P.R.s (containing 2-fold rotation axes) the search of which was exhausted previously.

(b) Determination and generation of the lattices could be pursued independently of the surfaces.

(c) Form generation of the lattices, on the E.P.R and on the periodic complex levels, is much simpler than that of the hyperbolic (minimal or otherwise) surfaces. The last require much more sophisticated and rigorous mathematical tools and intense computer power for their solution and graphical representation.

(d) Once we leave the domain of "simple" surfaces (those which could be generated as simulation of the dipping process of the 2-fold axes perimeters, as described), and approach the more complex ones, there is no way to generate them, on any level, without going through the lattice determination first, at least on the E.P.R level.

We have to remind ourselves that generating (additional) S.D.L.s and P.H.S.s is based on some already proven preconceptions:

The number of E.P.R.s is finite and was exhaustively derived from the symmetry groups (230).

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*"Mutually dual and identical" is a double characteristic and it is essential. Just "identical" is not sufficient.

2. The number of combined systems of E.P.R.s and specific sets of 2-fold rotation axes is also finite and completely determined.

3. Every E.P.R. contains complete representation of all the phenomena taking place within the "periodic complex", and particularly, representation of the whole periodic space, its symmetry group, the 2-fold axes network, the two complementary (and identical) subspaces, the S.D.L. -pair characterizing them, as well as the partition surface (P.H.S) in between.

It follows that exploration of E.P.R.'s space and combinatorial search for the S.D.L. pairs and the associated surfaces within, can lead to generation of all the solutions for each specific E.P.R.'s space subdivisions, (as well as for the entire periodic space), into two complementary and identical subspaces, and therefore, to the generation of all the S.D.L. pairs and the associated P.H.S.s in between.

(These three parameters should be part of the notation system).

4. Sum length of the lattice edges per one E.P.R unit.

5. Number of lattice vertices per one E.P.R unit

6. Area of the basic minimal surface unit (within the E.P.R).

7. Total curvature of the a.m. surface units.

This paper intends to report on a research in which generation of dual space lattices and the hyperbolic partition surfaces are in the core of the inquiry. It is evident from the report that the research is still in it's initial stages, concerned with the new evolving concepts, scope definitions and methodologies. But already at this stage, the impression is that the number of the solutions of mutually dual and identical lattice pairs and the associated periodic hyperbolic surfaces - partitions which subdivide the entire space into two complementary and identical subspaces, is infinite, but can accept the discipline of hierarchical typology and systematic classification.

[I] M. Burt A. Korren - Periodic hyperbolic surfaces and subdivision of 3-space. Hyperspace, vol. 3, #3, Dec. 1994 - Japan.

[II] M. Burt - Saddle polyhedra and close packing. Zodiac - 22, Apr, 1973 - Italy.

[III] A. Korren - Periodic 2-manifolds surfaces which divide the space into two identical subspaces. Master thesis, Tecnion, 1993 - Israel.